（入試演習）　三角関数 ②

問題 $\dfrac{1}{\tan\dfrac{\pi}{24}}-\sqrt{2}-\sqrt{3}-\sqrt{6}$ は整数である。その値を求めよ。

解答 $\dfrac{\pi}{24}=\theta$ とすると　$2\theta=\dfrac{\pi}{12}=\dfrac{\pi}{3}-\dfrac{\pi}{4}$

よって　$\sin 2\theta=\sin\left(\dfrac{\pi}{3}-\dfrac{\pi}{4}\right)$

$$=\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$=\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}-\frac{1}{2}\cdot\frac{1}{\sqrt{2}}=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos 2\theta=\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$

$$=\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$=\frac{1}{2}\cdot\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

ゆえに　$\dfrac{1}{\tan\dfrac{\pi}{24}}=\dfrac{1}{\tan\theta}=\dfrac{\cos\theta}{\sin\theta}=\dfrac{2\cos^2\theta}{2\sin\theta\cos\theta}=\dfrac{1+\cos 2\theta}{\sin 2\theta}$

$$=\left(1+\frac{\sqrt{3}+1}{2\sqrt{2}}\right)\times\frac{2\sqrt{2}}{\sqrt{3}-1}=\frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$$

$$=\frac{(2\sqrt{2}+\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}=\frac{2\sqrt{6}+2\sqrt{3}+2\sqrt{2}+4}{3-1}$$

$$=\sqrt{6}+\sqrt{3}+\sqrt{2}+2$$

したがって　$\dfrac{1}{\tan\dfrac{\pi}{24}}-\sqrt{2}-\sqrt{3}-\sqrt{6}=2$